Worksheet 04 - Conditional Probability and Independence

Shawn Kim

***Directions**: Please upload a PDF to Gradescope that includes both your written responses and corresponding R code inputs/outputs (if requested) for each problem.*

Special Directions: Be sure to demonstrate the correct use of the conditional probability notation (and mathematical notation in general) in your work. When showing your work, clearly show your reasoning by entering all necessary algebra/calculations as text or inserting a clear well-cropped image of your work using an R chunk (as shown in a few places, including part c of problem 2). Be sure to use clear/correct probability notation throughout your work (ie do not only show the numbers involved in calculations).

Problem 1. With the monsoon season, we can have more cases of dengue fever, which is a mosquito-borne tropical disease caused by the dengue virus. Antibody tests are recommended during a dengue outbreak. However, the presence of other viruses in the human body can have cross-reactive results, yielding a high false positive rate.

- Assume a false positive rate of 10% (i.e., $P\{\text{Test+} \mid \text{no dengue}\} = 0.10$)
- Assume a false negative rate of 1% (i.e., $P\{\text{Test-} \mid \text{has dengue}\} = 0.01$))

Problem 1 Part a) Given that a person has dengue, what is the probability of a positive test?

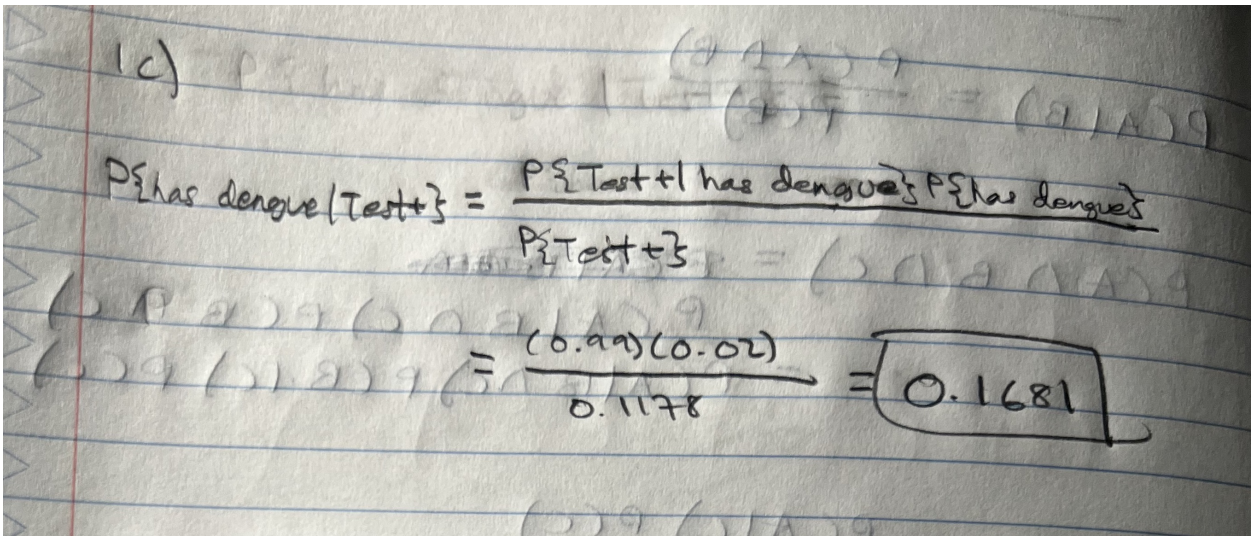
NOTE: As with many parts on this worksheet: Calculate this probability by hand, and demonstrate the correct use of the conditional probability notation in your work. Enter all necessary calculations as text or insert an image of your work using an R chunk.
P{Test+|has dengue} = 1 - P{Test-|has dengue} = 1 - 0.01 = 0.99

Problem 1 Part b) If two percent of a population has dengue (i.e., $P\{\text{has dengue}\} = 0.02$), what fraction of the population will test positive?

NOTE: Enter all necessary calculations as text or insert an image of your work using an R chunk.
P{Test+} = P{Test+|has dengue}P{has dengue} + P{Test+|no dengue}P{no dengue} = (0.99)(0.02) + (0.10)(0.98) = 0.1178

Problem 1 Part c) If the individual tests positive, what is the probability that this individual has dengue? In other words, determine $P\{\text{has dengue} \mid \text{Test+}\}$. Assume the same population dengue rate as in part (b) (i.e., $P\{\text{has dengue}\} = 0.02$).

NOTE: Upload an image of your work by replacing "upload_image.jpg" with your well-cropped appropriately titled .jpg file in the R chunk below.



Problem 1 Part d) Produce a two-way table showing the numbers of people with and without dengue and with and without a positive test out of a population of 10,000. Round each entry to the nearest integer. Assume the same population dengue rate as in part (b) (i.e., $P\{\text{has dengue}\} = 0.02$).

NOTE: To complete this task, fill in the table below by replacing each ? with its appropriate numerical value. You do not need to include the calculation for each entry on this one.

Population	no dengue	has dengue	Total
Test +	980	198	1178
Test -	8820	2	8822
Total	9800	200	10,000

Problem 1 Part e) Confirm your answer in part (c) by calculating the corresponding proportion using the information from the two way table in part (d).

NOTE: Enter all necessary calculations as text.

$P\{\text{has dengue} \mid \text{Test+}\} = (198)/(980+198) = 0.1681$